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# Wave Propagation in Strongly Coupled Quasi-One-Dimensional Quantum Plasma in a Magnetic Field

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A cold electron gas fills the lowest Landau level for superstrong magnetic fields and very low densities. In such cases, in general, the potential energy of the particles is equal to or greater than their kinetic energy (strongly coupled plasmas), and a special approach is called for. The STLS (Singwi, Tosi, Land, and Sjolander) approximation scheme is used to study the dispersion and damping of the low-frequency modes, i.e., the whistler and the extraordinary modes for zero temperature. The lowest order dispersion for all modes in consideration are unaffected by correlations, but for undamped plasmas the correlation term is of the order $\gamma a^2 c^{-2}$. Further, $\delta(\omega^2)$ for the whistler mode becomes infinite at $\gamma = 3$; its behavior critically depends on the "filling number" $\eta_{Fc} = \varepsilon_F/\hbar\Omega$, where $\varepsilon_F$ is the Fermi energy and $\Omega$ is the electron cyclotron frequency.

## 1. INTRODUCTION

In weakly coupled plasmas the potential energy of the particles is much less than their kinetic energy. Thus, the contribution of the potential energy of the particles is generally neglected when considering their response to excitation due to external or self-consistent fields. In the case, however, where the potential energy of the particles is equal to or greater than their kinetic energy (strongly coupled plasmas), a special approach is called for. In this work I consider a nonrelativistic homogeneous one-component quantum plasma in equilibrium in the presence of a uniform external magnetic field.

As in the classical plasma case (Carini *et al.*, 1980; Genga, 1988*a*), we use the STLS (Singwi, Tosi, Land, and Sjolander) method to study the dispersion and damping of the low-frequency modes, i.e., the whistler mode

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and the extraordinary mode for zero temperature. In the latter case, one has to distinguish between the nonresonant and the "resonant" situations, depending upon whether the cutoff frequency $\omega_1$ is different from or coincides with the electron cyclotron frequency $\Omega$. I consider all these modes when the direction of propagation is parallel, perpendicular, or at an oblique angle to the magnetic field. It is known (Carini *et al.*, 1980) that for strong coupling, the longitudinal plasmon exhibits negative dispersion. The main interest in studying transverse modes lies in determining whether a similar change takes place and what the critical coupling is. This problem is related to the Malmberg-O'Neil experiment, where a strongly magnetized, strongly coupled electron plasma is being generated. A further feature of the experiment is that because of the strong magnetic field, quantum effects are very important (Canuto and Ventura, 1972; Genga, 1988*b*). In this work I consider only spinless particles in the lowest Landau level at $T = 0$ K; since in the Malmberg-O'Neil experiment, the magnetic field is strong enough to limit the population of the particles to the lowest Landau level, I limit the calculation to this case. This superstrong magnetic field also gives rise to a one-dimensional quantum plasma (Canuto and Ventura, 1972; Genga, 1988*b*).

The approach used to study this problem is through a perturbation analysis of the plasma dispersion relations (Genga, 1988*a*, *b*). Then, by solving the dispersions for $\delta_u\omega$, the shift of frequency due to correlation for each mode in consideration, at different angles of propagation, all the needed information is obtained. $\delta_u\omega$ is of the order $k^2$; thus, it is small as $k \to 0$, which is equal to the order of frequency shifts due to refractive ($\delta_n\omega$), thermal ($\delta_t\omega$) and quantum ($\delta_q\omega$), effects, respectively, even for $\gamma \gg 1$ (Genga, 1986, 1988*a*,*b*). In Section 2, I determine the frequency shift due to correlations of the whistler and the extraordinary modes for arbitrary direction of propagation as a function of the coupling parameter $\gamma$ for plasmas without damping; damping effects on these modes are considered in Section 3.

## 2. PLASMAS WITHOUT DAMPING

It is known (Genga, 1988*a*) that for propagation parallel to a magnetic field there are no coupling effects on the transverse modes; thus, we recover the results of the weakly coupled plasmas (Genga, 1988*b*). It is also known that the whistler mode and the resonant situation of the extraordinary mode do not exist for propagation perpendicular to a magnetic field. These results are also valid for plasmas with damping considered in Section 3.

## 2.1. Whistler Mode

After a small perturbation is applied on the plasma dispersion relations, the frequency shift obtained in this case is given by

$$
\begin{aligned}
\delta(\omega^2) = \frac{-\Omega^2 k^6 c^6}{\omega_p^8} & \left\{ \left[ 1 + \frac{\omega_p^2}{\Omega^2}\left(1 + \frac{\tan^2\theta}{32 n_{\mathrm{Fc}}^2}\right)\right] \cos^2\theta \right. \\
& + \left(1 + \frac{\omega_p^2}{\Omega^2}\right) - \frac{\omega_p^2}{32\Omega^2}\left(51 - \frac{1}{2n_{\mathrm{Fc}}}\right)\frac{\sin^2\theta}{n_{\mathrm{Fc}}} \\
& + \frac{\omega_p^4 a^2}{2\Omega^2 c^2}\left\{ \left[ 1 + \frac{\omega_p^2}{\Omega^2}\left(1 + \frac{\tan^2\theta}{32 n_{\mathrm{Fc}}}\right)\right] \right. \\
& \times \left[ \left(1 - \frac{4n_{\mathrm{Fc}}}{3}\right)(1 + 2\cos^2\theta)\cos^2\theta \right. \\
& \left. \left. - 2n_{\mathrm{Fc}}\left(1 - \frac{33}{64 n_{\mathrm{Fc}}} + \frac{1}{16 n_{\mathrm{Fc}}^2}\right)\sin^2\theta\cos^2\theta \right\} \right. \\
& + \left(1 + \frac{\omega_p^2}{\Omega^2}\right)\left[ 3\left(1 - \frac{4n_{\mathrm{Fc}}}{3}\right)\cos^2\theta + \frac{1}{32}\left(17 - \frac{3}{n_{\mathrm{Fc}}}\right)\sin^2\theta \right. \\
& + 4[(1 - 2n_{\mathrm{Fc}})\cos^2\theta + \tfrac{3}{4}\sin^2\theta] \\
& - \frac{\gamma}{3}\left\{ \left[ 1 + \frac{\omega_p^2}{\Omega^2}\left(1 + \frac{\tan^2\theta}{32 n_{\mathrm{Fc}}^2}\right)\right] \right. \\
& \times \left[ \left(1 - \frac{4n_{\mathrm{Fc}}}{3}\right)(4 + 5\cos^2\theta)\cos^2\theta - 8n_{\mathrm{Fc}} \right. \\
& \left. \times \left(1 - \frac{1}{64 n_{\mathrm{Fc}}} + \frac{1}{16 n_{\mathrm{Fc}}^2}\right)\sin^2\theta\cos^2\theta \right] \\
& + \left(1 + \frac{\omega_p^2}{\Omega^2}\right)\left[ 9\left(1 - \frac{4}{3}n_{\mathrm{Fc}}\right)\cos^2\theta - 4n_{\mathrm{Fc}} \right. \\
& \left. \times \left(1 - \frac{33}{32 n_{\mathrm{Fc}}} + \frac{9}{64 n_{\mathrm{Fc}}^2}\right)\sin^2\theta + 12[(1 - 2n_{\mathrm{Fc}})\cos^2\theta + \tfrac{3}{4}\sin^2\theta] \right\} \\
& + \frac{\gamma^2}{9}\left\{ \left[ \frac{\omega_p^2}{1 + \Omega^2}\left(1 + \frac{\tan^2\theta}{32 n_{\mathrm{Fc}}^2}\right)\right]\left[ \left(1 - \frac{4n_{\mathrm{Fc}}}{3}\right)(5 + 4\cos^2\theta) \right. \right. \\
& - 3n_{\mathrm{Fc}}\left(4 - \frac{21}{16 n_{\mathrm{Fc}}} + \frac{1}{24 n_{\mathrm{Fc}}^2}\right)\sin^2\theta\cos^2\theta \\
& + \left(1 + \frac{\omega_p^2}{\Omega^2}\right)\left[ 9\left(1 - \frac{4n_{\mathrm{Fc}}}{3}\right)\cos^2\theta - n_{\mathrm{Fc}}\left(11 - \frac{63}{16 n_{\mathrm{Fc}}} + \frac{39}{8 n_{\mathrm{Fc}}^2}\right) \right.
\end{aligned}
$$

$$\times \sin^2\theta + 12\left[(1-2n_{Fc})\cos^2\theta + \tfrac{3}{4}\sin^2\theta - \frac{\gamma^3}{27}\left\{\left[1+\frac{\omega_p^2}{\Omega^2}\left(1+\frac{\tan^2\theta}{32n_{Fc}^2}\right)\right]\right.\right.$$
$$\times\left[\left(1-\frac{4n_{Fc}}{3}\right)(1+2\cos^2\theta)\cos^2\theta\right.$$
$$\left.\left.-8n_{Fc}\left(1-\frac{9}{64n_{Fc}}+\frac{1}{32n_{Fc}^2}\right)\sin^2\theta\cos^2\theta\right\}\right.$$
$$+\left(1+\frac{\omega_p^2}{\Omega^2}\right)\left\{3\left(1-\frac{4n_{Fc}}{3}\right)\cos^2\theta - n_{Fc}\left(6-\frac{13}{8n_{Fc}}+\frac{7}{32n_{Fc}^2}\right)\sin^2\theta\right.$$
$$\left.\left.+4[(1-2n_{Fc})\cos^2\theta+\tfrac{3}{4}\sin^2\theta]\right\}\right\}\left[1-\gamma+\frac{\gamma^2}{3}-\frac{\gamma^3}{27}\right]^{-1} \tag{1}$$

For $\theta = 0^0$, the $\gamma$ terms vanish in equation (1) and we obtain the equation for weakly coupled plasmas, Figure 1 shows that $\delta_u(\omega^2)$ is $\gamma$, $n_{Fc}$, and angle dependent. At $\gamma = 3$, equation (1) is infinite.

### 2.2. Nonresonant Case of Extraordinary Mode

For propagation parallel to the magnetic field the frequency shift after a small perturbation is applied on the dispersion relation is given by

$$\delta\omega = \frac{1}{2\omega_1+\Omega}\left[\frac{(\omega_1+\Omega)^2c^2}{2\omega_p^2}+\Omega^2a^2\left(\frac{\omega_1+\Omega}{\omega_1+2\Omega}+\frac{\gamma}{3}\eta_{Fc}\right)\right]k^2 \tag{2}$$

where $a^2 = \hbar/M\Omega$. Correlations enhance the positive quantum dispersion.

Turning to an oblique propagation, we find that

$$\delta\omega = \frac{1}{2(2\omega_1+\Omega)}\left\{\frac{(\omega_1+\Omega)^2c^2}{\omega_p^2}(1+\cos^2\theta)+\Omega^2a^2\left[\left(1-\frac{4}{3}\frac{\Omega}{\omega_1+\Omega}\eta_{Fc}\right)\cos^2\theta\right.\right.$$
$$\left.\left.+2\left(\frac{\omega_1+\Omega}{\omega_1+2\Omega}+\frac{\gamma}{3}\eta_{Fc}\right)\sin^2\theta\right]\right\}k^2 \tag{3}$$

As in the parallel propagation case, correlations increase positive quantum dispersion. Figure 2 shows that $\delta_u\omega$ is positive and angle dependent.

### 2.3. Resonant Case

As in the case of classical plasmas (Genga, 1988*a*), the elements of the strongly coupled dielectric tensor are obtained by assuming that

$$u(\alpha_{22}^0\sin^2\theta + 2\alpha_{23}^0\sin\theta\cos\theta + \alpha_{33}^0\cos^2\theta) \ll 1$$

where $\alpha_{22}^0$, $\alpha_{23}^0$, and $\alpha_{33}^0$ are weakly coupled polarizabilities. The correlational terms are also found to be of order $k^2$ smaller than those of the correlationless one; hence, their contributions are negligible for the

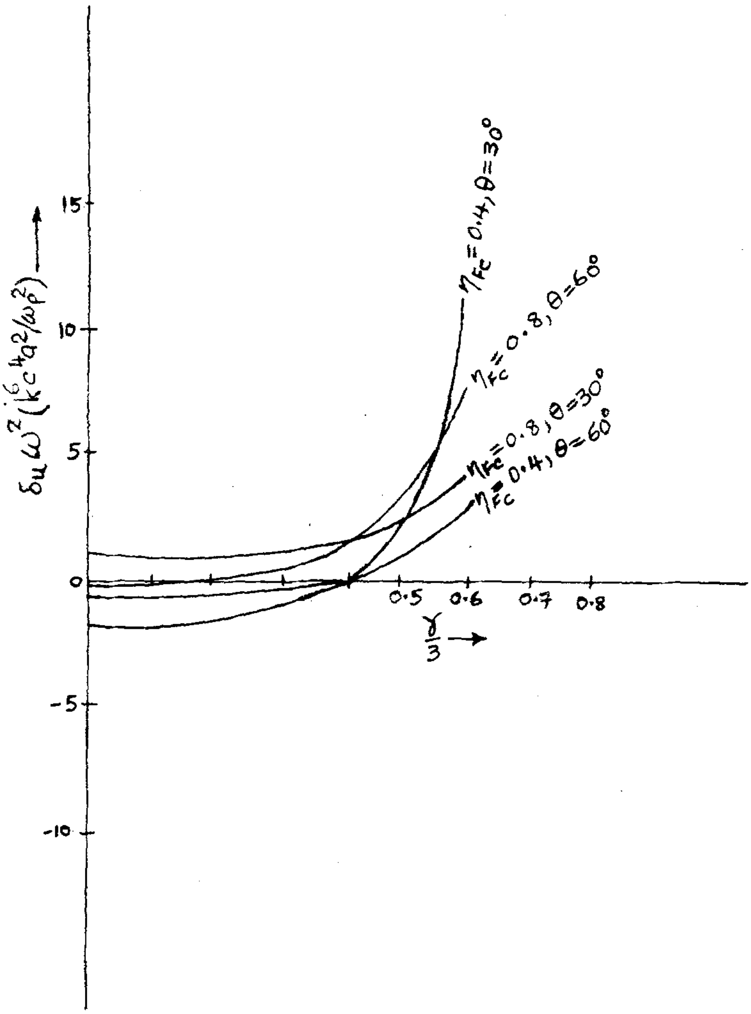


**Fig. 1.** Whistler mode: strong coupling terms $\delta_u\omega^2$ (in units of $k^6c^4a^2/\omega_p^2$) versus $\gamma/3$ for $\eta_{Fc}=0.4, 0.8$; $\theta=30°, 60°$.

frequency shift of order $k^2$. However, for frequency shift of order greater than or equal to $k^4$ their contributions are considered.

## 3. PLASMAS WITH DAMPING

As in the case of weakly coupled quantum plasmas (Genga, 1988*b*), we find that the strongly coupled whistler mode and the nonresonant

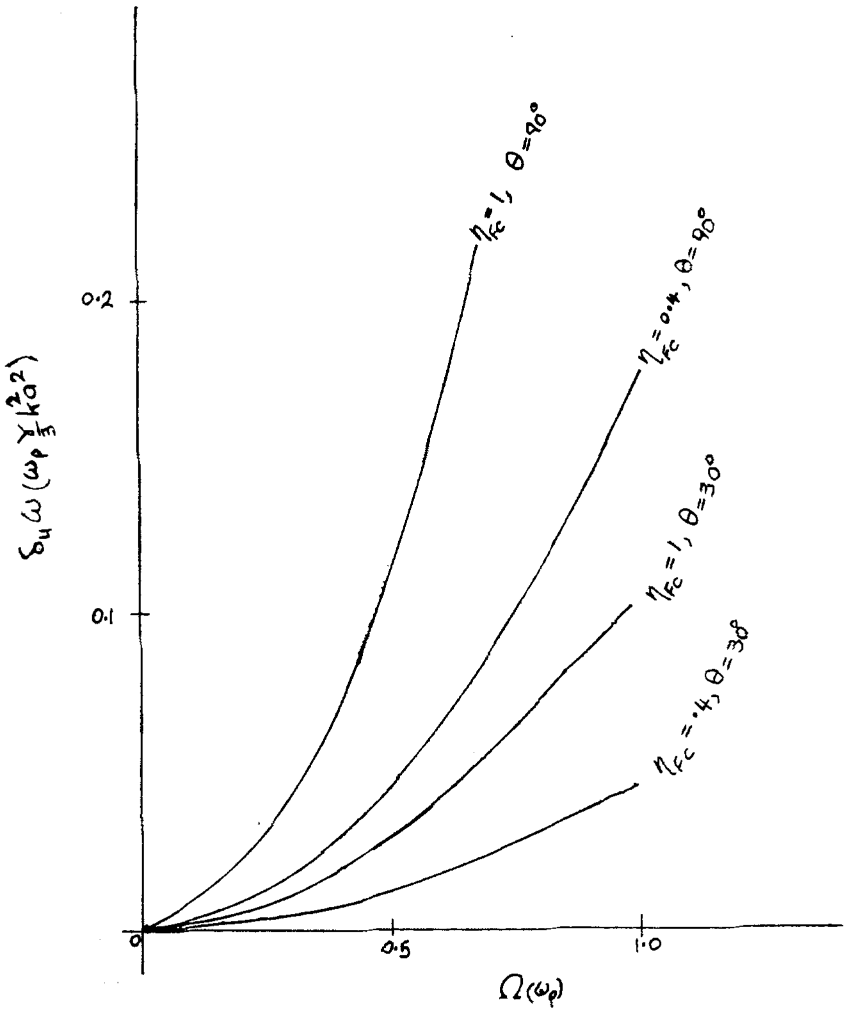


**Fig. 2.** Extraordinary mode: strong coupling term $\delta_u\omega$ [in units of $\omega_p(\gamma/3)k^2a^2$] versus $\Omega$ (in units of $\omega_p$) for $\eta_{Fc}=0.4$, 1.0; $\theta=30°, 90°$.

situation of the extraordinary mode are unaffected by damping; hence, the results of the strongly coupled plasmas without damping are recovered for each of these cases.

For the resonant situation of the extraordinary mode, the correlational terms of components of the polarizability tensor are of order $k^2$ smaller than the correlationless ones, as in the case of strongly coupled plasmas without damping. This means that for the frequency shift of order $k^2$ the contribution of the correlational term is negligible. However, for frequency shifts of order greater than or equal to $k^2$, the correlational term contribution has to be considered. Since I consider only the harmonic frequency shift of order $k^2$, the results of the weakly coupled plasmas with damping are recovered.

## 4. CONCLUSION

I have shown that the lowest order dispersion relations for all the modes in consideration are again (Genga, 1988*a*) unaffected by correlations. Further, for plasmas without damping, correlations are of the order $\gamma a^2 c^{-2}$. Hence, in order for the correlations to be noticeable, $\gamma$ has to be of the order $a^{-2}c^2$. Finally at $\gamma = 3$, $\delta(\omega^2)$ for the whistler mode blows up, for the same reason as that for warm classical plasmas without damping (Genga, 1988*a*).

## REFERENCES

Canuto, V., and Ventura, J. (1972). *Journal of Astrophysics and Space Science*, **18**, 104.

Carini, P., Golden, K. I., and Kalman, G. (1980). *Physical Review Letters*, **78A**, 450.

Genga, R. O. (1986). Thermal effects on plasma dispersion in a magnetic field presented at the "First Regional Seminar in Physics—Physics for Development," Abidjan, Ivory Coast, (19-21 March, 1986).

Genga, R. O. (1988*a*). Wave propagation in strongly coupled classical plasmas in a magnetic field, *International Journal of Theoretical Physics*, **27**, 649.

Genga, R. O. (1988*b*). Wave propagation in quasi one-dimensional quantum plasma in a magnetic field, *International Journal of Theoretical Physics*, **27**, 835.